Section 2.4

The Chain Rule: If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x, then $y = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x).$$

1) Each of the following is a composite function of the form $y = f(g(x))$. Identify $u = g(x)$ and $y = f(u)$ in each case.

   a) $y = \sqrt{x^2 - 3x}$

   b) $y = \sin(x^2)$

   c) $y = \frac{3}{2x^2 - 1}$

   d) $y = \sin^3 x$

The General Power Rule: If $y = [u(x)]^n$, where u is a differentiable function of x and n is a rational number, then

$$\frac{dy}{dx} = n[u(x)]^{n-1}\frac{du}{dx}$$

or, equivalently,

$$\frac{d}{dx}[u^n] = nu^{n-1}u'.$$

2) Find the derivative of the following functions.

a) $f(x) = (5x^3 - 4x)^6$

b) $g(x) = \frac{3}{8x^2 - 1}$

3) Find all points on the graph of $f(x) = \sqrt[3]{x^2 - 4}$ for which

a) $f'(x) = 0$

b) $f'(x)$ does not exist.

4) Find and simplify the derivatives of the following functions.

a) $f(x) = 2x^3\sqrt[3]{3x^2 - 4}$

b) $g(x) = \frac{2x-1}{\sqrt{3x^2-1}}$

c) $h(x) = \left(\frac{5x^3-2}{2x+3}\right)^3$

5) Find the derivatives of the following functions.

a) $y = \sin^2 x$

b) $y = \tan x^2$

c) $f(x) = 2\cos^2 3x$

d) $g(\theta) = 3\theta \sec 2\theta$

e) $h(t) = \sin(\cos t)$

6) Find an equation of the tangent line to $f(x) = \cos 2x - 3\sin x$ at the point $\left(\frac{\pi}{6}, -1\right)$. Then determine all the values of x in the interval $(0, 2\pi)$ at which the graph has a horizontal tangent.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #3, 27, 32, 59, 79, 89, 103